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The local Poisson hypothesis for solar flares

M.S. Wheatland

School of Physics, The University of Sydney, NSW 2006, Australia

wheat@physics.usyd.edu.au

ABSTRACT

The question of whether flares occur as a Poisson process has important consequences for flare physics. Recently Lepreti et al. presented evidence for local departure from Poisson statistics in the Geostationary Operational Environmental Satellite (GOES) X-ray flare catalog. Here it is argued that this effect arises from a selection effect inherent in the soft X-ray observations; namely that the slow decay of enhanced flux following a large flare makes detection of subsequent flares less likely. It is also shown that the power-law tail of the GOES waiting-time distribution varies with the solar cycle. This counts against any intrinsic significance to the appearance of a power law, or to the value of its index.

Subject headings: Sun: activity — Sun: flares – Sun: corona – Sun: X-rays

1. Introduction

There has been increased recognition of the importance of flare statistics for understanding the mechanisms of energy storage and release in the solar corona. The distribution of times Δt between events ("waiting times") provides information on whether flares occur as independent events, and also represents a test for certain flare models. In particular, the avalanche model for flares (Lu and Hamilton 1991; Lu et al. 1993) and variants thereof, predict that flares are independent and occur with a constant rate, and hence obey Poisson statistics. The waiting-time distribution (WTD) for a Poisson process with a rate λ is an exponential,

$$P(\Delta t) = \lambda e^{-\lambda \Delta t}. \tag{1}$$

Observational determinations of the flare WTD based on different data sets have given a variety of results (see e.g. Wheatland et al. 1998 for a brief review). Boffeta et al. (1999) showed that the WTD constructed from the GOES catalog of 20 years of flaring exhibits a power-law tail. They argued that the appearance of a power law contradicts the avalanche model prediction of Poisson

statistics, and supports instead an MHD turbulence model of the flaring process. Special significance was attributed to the value of the power-law index ($\alpha \approx -2.4$) derived from the observations. In response, Wheatland (2000) argued that over the course of the several solar cycles included in the GOES data, the flaring rate varies by more than an order of magnitude, and so flares cannot be assumed to be occurring at a constant rate. If the flaring process can be represented by a piecewise constant Poissson process with constant rates λ_i for intervals t_i, then the WTD may be described by

$$P(\Delta t) = \sum_i \varphi_i \lambda_i e^{-\lambda_i \Delta t}, \tag{2}$$

where $\varphi_i = \lambda_i t_i / \sum_i \lambda_i t_i$ is the fraction of events corresponding to the rate λ_i. Rates λ_i and intervals t_i were estimated for the GOES catalog events using a Bayesian procedure. The resulting WTD [equation (2)] was shown to qualitatively reproduce the observed power-law tail. The GOES catalog involves flares occurring in all active regions on the Sun. Wheatland (2001) extended the analysis to flares in individual active regions, and showed that equation (2) accounts quantitatively for the observed WTDs in a number of very flare-productive active regions.

Recently Lepreti et al. (2001) responded by showing that a statistical test rejects the hypothesis that the GOES catalog events are locally Poisson. They showed that the flare WTD can be fitted by a Lévy function, and argued that this reflects the existence of memory in the system and once again supports a turbulence model for the flare process.

Although the question of whether flares occur as a Poisson process may seem arcane, the consequences for the understanding of the flare process are substantial.

In this paper the result of Lepreti et al. (2001) is re-examined. In § 2 an alternative explanation for the local departure of the GOES flares from Poisson statistics is given, namely that it arises from the failure to detect flares occurring soon after large flares because of the increase in soft X-ray flux associated with the large flare. The nature of soft X-ray data and the event selection procedure used to compile the GOES catalog makes this effect inevitable. In § 3 it is shown that the power-law tail of the GOES WTD varies with the solar cycle. This result argues against any particular significance to the value of the power-law index of the tail of the WTD. Finally in § 4 the consequences of these results to our understanding of flares are discussed.

2. Local departure from Poisson statistics

Lepreti et al. (2001) applied a test for local Poisson behaviour devised by Bi et al. (1989) to the GOES catalog events. Flares are replaced by their peak times. Let X_i denote the time interval between the peak time of event i and that of its nearest neighbour. Also, let Y_i denote the time interval to the *other* neighbour. Under the local Poisson hypothesis, X_i and Y_i have probability

distributions $P(X_i) = 2\lambda_i \exp(-2\lambda_i X_i)$ and $P(Y_i) = \lambda_i \exp(-\lambda_i Y_i)$. In this case the variable

$$H_i = \frac{X_i}{X_i + \frac{1}{2}Y_i} \tag{3}$$

should be uniformly distributed between zero and one, and then the observed distribution of H_i provides a test of the local Poisson hypothesis. Lepreti et al. (2001) reported a significantly non-uniform distribution of H_i for the GOES events. We have repeated the procedure for GOES events between 1981-1999 above C1 class (for details of the GOES data, see Wheatland 2001), and the result is shown in Fig. 1. The solid curve is the cumulative distribution of H_i. A Kolmogorov-Smirnov test rejects the hypothesis that the distribution of H_i is uniform, confirming the finding of Lepreti et al. (2001).

Of course, the departure of the GOES catalog events from Poisson behaviour does not necessarily mean that solar flares are not locally Poisson. It is important to consider how the catalog was constructed, and in particular any selection biases that might introduce a departure from Poisson statistics. Recently Wheatland (2001) identified such an effect, which was termed *obscuration*. The GOES detectors record soft X-ray light curves for flares, which are characterised by relatively fast rises to a peak flux, followed by slow decays in flux due to the cooling of hot plasma in the corona produced by the flare (Feldman 1996). Large flares produce enhancements of more than a factor of 100 in flux, and this increase may take many hours to decline. During the decay time subsequent flares may be missed because of the enhanced background. The selection procedure used to compile the GOES catalog imposes constraints on the detection of flares. The start of a flare is defined by four one-minute flux values that are monotonically increasing, with the final flux being at least 1.4 times the first. For flares occurring shortly after a large flare, this implies that the second flare must produce an increase in flux at least of order 40% of the flux of the first flare to be detected. Even quite significant events can be missed in the wake of a large flare. Since flares follow a power-law peak flux distribution (Hudson 1991), the majority of flares are small and a large number of flares are expected to be missing from the catalog as a result. Wheatland (2001) estimate that in the absence of obscuration the number of flares above C1 class would be higher by $(75 \pm 23)\%$.

The following simulation confirms that obscuration can account for the observed departure from Poisson statistics. Using the Bayesian decomposition of the GOES catalog into rates λ_i and intervals t_i described in Wheatland (2000), a sequence of flare times was generated as a piecewise constant Poisson process. As a check, the Bi et al. (1989) test was applied to this sequence of times, and local Poisson behaviour was confirmed (the cumulative distribution of H_i is shown by the dotted line in Fig. 1). Next, a decay time was assigned to each synthetic flare, based on random selection from the decay times observed for the real GOES events. To mimic obscuration, any events falling within the decay time of another event were excluded from the sequence. The Bi et al. (1989) test was applied to the resulting sequence of flares. The result is shown as the dashed curve in Fig. 1. Once again the distribution of H_i is significantly different from that expected from local Poisson behaviour. The distribution is clearly very similar to that found for the GOES data. Although this simulation is crude, it shows that a Poisson simulation with obscuration produces

an effect mimicking that reported by Lepreti et al. (2001) for the real GOES flares.

3. Solar cycle variation

Boffeta et al. (1999) argued that the observed power-law tail in the WTD for the GOES events is "robust," and even claimed that the power law in waiting times is "as firmly established as the power laws observed for total energy, peak luminosity, and time duration." Lepreti et al. (2001) went further by assigning special significance to the fitting of the power law by a Lévy function. The value of the power-law index is important to this fit.

A feature of the other power laws (energy, peak flux, and possibly duration) observed for solar flares is that they *do not vary substantially with the 11-year solar cycle* (Dennis 1985; Lu and Hamilton 1991), although evidence has been presented for small variations (Bai 1993). Here we show that, by contrast, the power law in waiting times is strongly dependent on the phase of the cycle.

Fig. 2 shows the WTDs for the maximum and minimum phases of the solar cycle. Maximum phase has been defined as times when the smoothed monthly sunspot number[1] is above 125, and minimum has been defined as times when the sunspot number is below 30. These definitions are arbitrary, but they provide an objective choice of cycle phase, i.e. they are made without reference to the flare data. The two histograms are clearly different. The histogram extending to larger waiting times corresponds to minimum phase. This is not surprising: the mean rate of flaring varies by more than an order of magnitude over the cycle, and since the mean rate is the reciprocal of the average waiting time, it follows that the WTD must vary with the cycle. Fits to the power-law tails ($> 10\,\mathrm{h}$) of the WTDs shown in Fig. 2 give power-law indices of -3.2 ± 0.2 and -1.4 ± 0.1 for the maximum and minimum phases respectively. The solid curves shown in Fig. 2 correspond to the piecewise constant Poisson model (2) with rates and times obtained by the Bayesian procedure (Wheatland 2000). This model accounts for the qualitative shape of the WTDs in each case, although clearly there is some discrepancy, which is most likely due to the failure of the Bayesian procedure to account for all of the rate variation.

4. Conclusions

Lepreti et al. (2001) demonstrated a departure from Poisson statistics in the GOES catalog of flares, and attributed this to "time invariance" in the flaring history. Here an alternative explanation is given, namely that the flux increase due to large flares reduces the likelihood of detecting subsequent smaller flares. This selection effect (termed obscuration) introduces a departure from Poisson

[1] This data, as well as the GOES catalog, is available from ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA.

statistics because it effectively means that the rate decreases when a large flare occurs, and hence the flares in the GOES catalog are not strictly independent. A simulation confirms that obscuration produces an effect very similar to that reported by Lepreti et al. (2001). This study underlines the need to consider all potential biases in the statistical study of flares.

The power-law tail to the WTD constructed from the GOES catalog has been shown to vary with time, and in particular with the solar cycle. On this basis it is difficult to argue that the power law is "robust," in marked contrast with the other power laws reported in flare statistics.

The picture emerging from this study and others (e.g. Wheatland 2001) is that flares occur as a time-varying, or non-stationary Poisson process. The observed WTD depends on the observed rates of flaring (Wheatland 2000) which naturally accounts for the variation of the WTD over the solar cycle. There is no particular significance to the appearance of a power law in the WTD. Indeed, adopting the continuous version of (2), viz.

$$P(\Delta t) = \frac{1}{\lambda_0} \int_0^\infty f(\lambda)\lambda^2 e^{-\lambda \Delta t}\, d\lambda, \tag{4}$$

where $f(\lambda)d\lambda$ is the fraction of time that the rate is in the range λ to $\lambda + d\lambda$ and $\lambda_0 = \int_0^\infty \lambda f(\lambda)\, d\lambda$ is the mean rate of flaring, the asymptotic behaviour of $P(\Delta t)$ is obtained by Taylor expanding $f(\lambda)$ about $\lambda = 0$. This gives

$$P(\Delta t) = \frac{2f(0)}{\lambda_0}(\Delta t)^{-3} + \frac{6f'(0)}{\lambda_0}(\Delta t)^{-4} + ... \tag{5}$$

Equation (5) suggests that the asymptotic form of $P(\Delta t)$ will always be a power law, with an index of -3. In assessing this result it should be remembered that (4) is an approximate expression (it assumes the rate does not varying greatly during a waiting time). Lepreti et al. (2001) appeared to forget this point in criticising the WTD obtained by Wheatland (2000) with an exponential form in (4).

Interestingly, Earthquake data also often exhibit power-law WTDs due to non-stationarity of Earthquake sequences. Utsu (1970) states that there is "no primary importance" to the appearance of a power law, but the power law exhibited by Earthquake size distributions (the Gutenberg-Richter relationship) is considered important. The situation seems to be closely analogous to that for flares.

The results of this paper appear to be consistent with an avalanche model of flaring subject to a time-varying rate of driving (Norman et al. 2001). However, the appearance of Poisson statistics does not exclude other models with similar general hypotheses concerning energy storage. Indeed, any steady state system with rates of downward transition in energy (flares) depending only on the size of the transition, and with a mean energy larger than the majority of flares, will exhibit Poisson statistics (Craig and Wheatland 2001).

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REFERENCES

Bai, T. 1993, ApJ 404, 805

Bi, H., Börner, G., and Chu, Y. 1989, A&A 218, 19

Boffeta, G., Carbone, P., Giuliani, P., Veltri, P., and Vulpiani, A. 1999, Phys. Rev. Lett. 83, 22

Craig, I.J.D. and Wheatland, M.S. 2001, in preparation

Dennis, B.R. 1985, Sol. Phys. 100, 465

Feldman, U., 1996, Physics of Plasmas 3, 3203

Hudson, H., 1991, Sol. Phys. 133, 357

Lepreti, F., Carbone, V., and Veltri, P. 2001, ApJ 555, L133

Lu, E.T., and Hamilton, R.J. 1991, ApJ 380, L89

Lu, E.T., Hamilton, R.J., McTiernan, J.M., and Bromund, K. 1993, ApJ 412, 841

Norman, J.P., Charbonneau, P., McIntosh, S.M., and Liu, H.-L. 2001, ApJ 557, yet to appear

Utsu, T. 1970, Journal of the Faculty of Science, Hokkaido University, Series VII (Geophysics), Vol. III, No. 4, 197

Wheatland, M.S. 2000, ApJ 536, L109

Wheatland, M.S. 2001, submitted to Sol. Phys.

Wheatland, M.S., Sturrock, P.A., and McTiernan, J.M. 1998, ApJ 509, 448

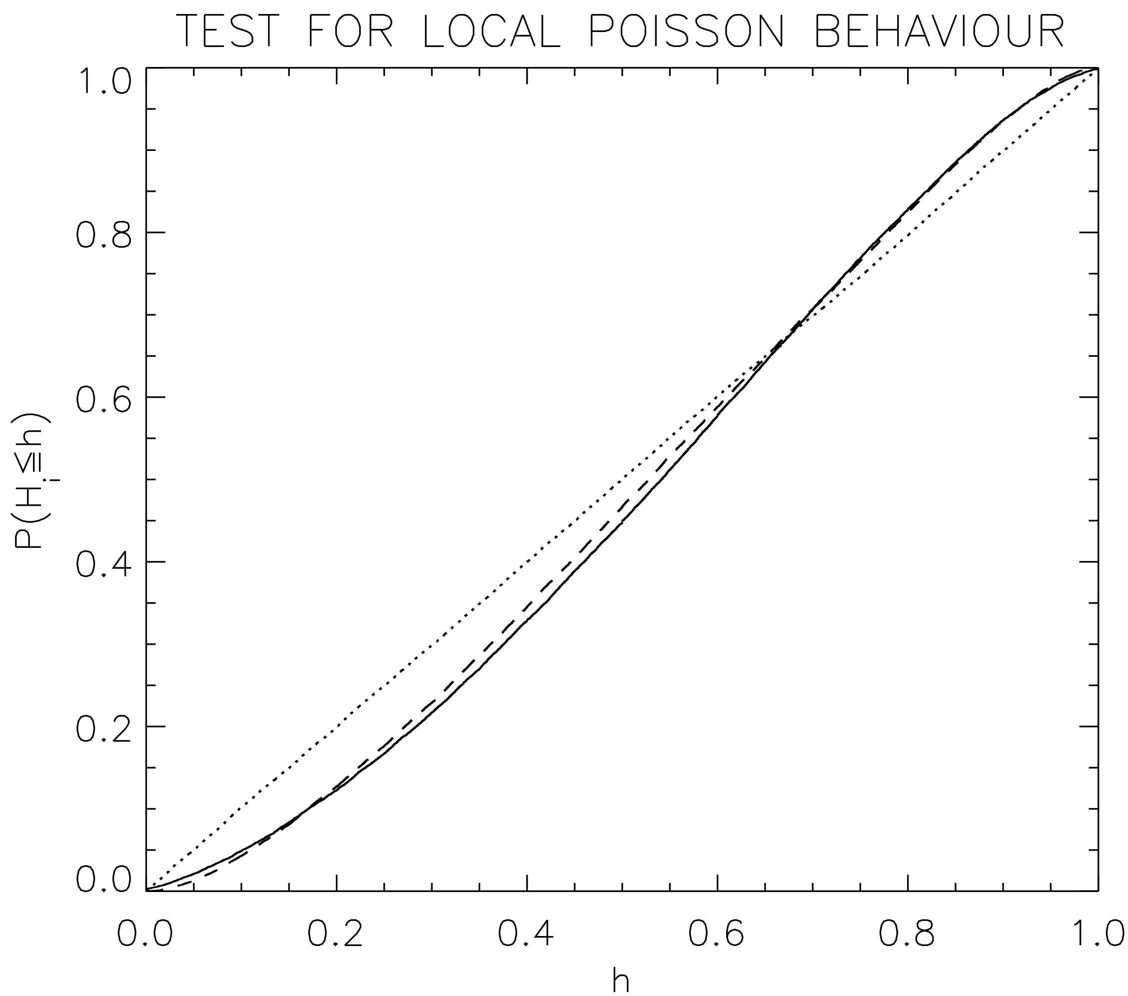

Fig. 1.— The Bi et al. (1989) test for Poisson behaviour, applied to the GOES data (solid), to a Poisson simulation (dotted), and to the same Poisson simulation including obscuration (dashed).

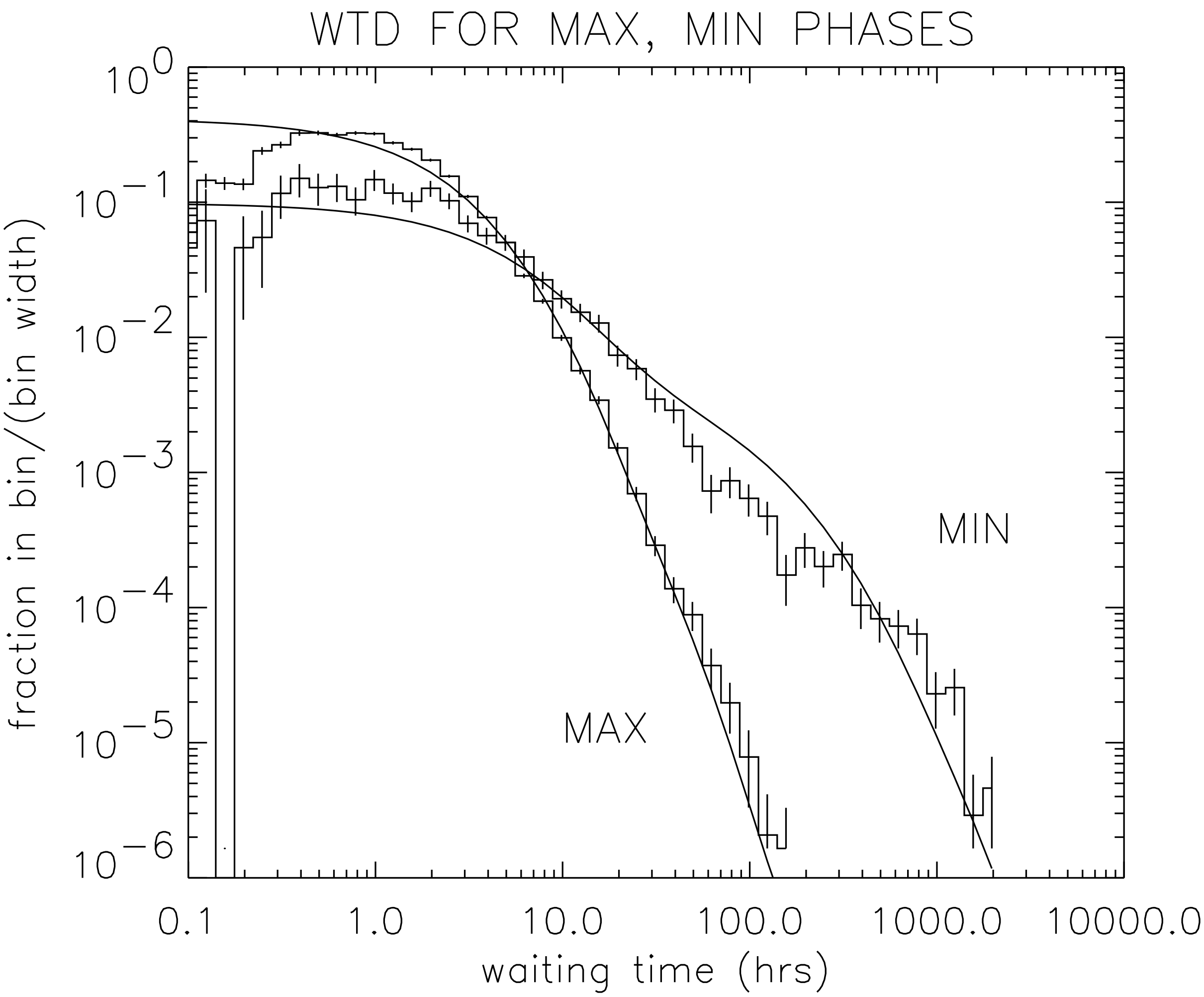

Fig. 2.— WTDs for the maximum and minimum phases of the solar cycle.